

Improved analysis on the semi-leptonic decay $\Lambda_c \rightarrow \Lambda \ell^+ \nu$ from QCD light-cone sum rules

Yong-Lu Liu, Ming-Qiu Huang and Dao-Wei Wang

Department of Physics, National University of Defense Technology, Hunan 410073, China

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With the renewed distribution amplitudes of Λ , we present a reanalysis on the semi-leptonic decay $\Lambda_c \rightarrow \Lambda \ell^+ \nu$ by use of the light-cone sum rule approach with two kinds of interpolating currents. The form factors describing the decay process are obtained and used to predict the decay width. With the inclusion of up to twist-6 contributions the calculations give the decay width $\Gamma = (10.04 \pm 0.88) \times 10^{-14} \text{GeV}$ for Chernyak-Zhitnitsky-type(CZ-type) current and $\Gamma = (6.45 \pm 1.06) \times 10^{-14} \text{GeV}$ for Ioffe-type current. The Ioffe-type interpolating current is found to be better for the estimation of the decay rate from a comparison with experimental data.

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I. INTRODUCTION

Flavor changing decays of heavy hadrons are of great interest in the heavy flavor physics due to their ability to provide useful information on the quark structure of the hadrons and reveal the nature of the weak interactions. In particular, the decay of the c -quark baryons can give us various charm related Cabibbo-Kobayashi-Maskawa matrix elements, which are the main ingredients of the standard model. Furthermore, a thorough understanding of the standard model itself needs a comprehension of the flavor changing dynamics. However, such a comprehension is difficult contemporarily since form factors characterizing those processes are nonperturbative quantities that need to be determined by some nonperturbative method. This paper aims to give a preliminary determination of the form factors of the exclusive semi-leptonic decay $\Lambda_c \rightarrow \Lambda \ell^+ \nu$. In the calculation we will use the method of QCD sum rules on the light cone [1], which in the past has been successfully applied to various problems in heavy meson physics, see [2] for a review.

The light-cone sum rule (LCSR) is a non-perturbative method developed from the standard technique of the traditional QCD sum rules from Shifman, Vainshtein, and Zakhavov(SVZ sum rules) [3], which comes as the remedy for the conventional approach in which vacuum condensates carry no momentum [4]. The main difference between SVZ sum rule and LCSR is that the short-distance Wilson OPE(Operator Product Expansion) in increasing dimension is replaced by the light-cone expansion in terms of distribution amplitudes (DAs) of increasing twist, which were originally used in the description of the hard exclusive process [5]. In recent years there have been many applications of LCSR to baryons. The nucleon electromagnetic form factors were studied for the first time in Refs. [6, 7] and later in Refs. [8, 9, 10] for a further consideration. Several nucleon related processes gave fruitful results within LCSR, the weak decay $\Lambda_b \rightarrow p \ell \nu_\ell$ was considered in both full QCD and HQET LCSR [11]. The generalization to the $N\gamma\Delta$ transition form factor was worked out in Ref. [12]. We have given the applications of LCSR on other $J^P = \frac{1}{2}^+$ octet baryons in Refs. [13, 14].

In this paper we will make use of the LCSR approach to study the decay process $\Lambda_c \rightarrow \Lambda \ell^+ \nu$, which has been preliminarily studied in the previous work [15]. The improvement of the present paper is to use the renewed distribution amplitudes provided in Ref. [13], and adopt two different kinds of interpolating currents to investigate the process. This transition had been studied in the literature by several authors, employing flavor symmetry or quark model or both in Refs. [16, 17, 18, 19, 20]. There are also QCD sum rule description of the form factors [21], upon which the total decay rate are obtained.

The paper is organized as follows: In Sec. II, we present the relevant Λ baryon DAs. Following that Sec. III is devoted to the LCSRs for the semi-leptonic $\Lambda_c \rightarrow \Lambda \ell^+ \nu$ decay form factors with two kinds of interpolating currents for the Λ_c baryon. The numerical analysis and our conclusion are presented in Sec. IV.

II. THE Λ BARYON DISTRIBUTION AMPLITUDES

The DAs presented in this subsection is the same as that in our previous work [15] and a part of the complete results in Ref. [13]. Herein we only give them out for the completeness of the paper. Our discussion for the Λ baryon DAs parallels with that for the nucleon in Ref. [22], so we just list the results following from that procedure and it is recommended to consult the original paper for details. The Λ baryon DAs are defined through the following matrix element:

$$4\langle 0|\epsilon_{ijk}u_{\alpha}^i(a_1x)d_{\beta}^j(a_2x)s_{\gamma}^k(a_3x)|P\rangle = \mathcal{A}_1(P\gamma_5 C)_{\alpha\beta}\Lambda_{\gamma} + \mathcal{A}_2M(P\gamma_5 C)_{\alpha\beta}(\not{x}\Lambda)_{\gamma} + \mathcal{A}_3M(\gamma_{\mu}\gamma_5 C)_{\alpha\beta}(\gamma^{\mu}\Lambda)_{\gamma} \\ + \mathcal{A}_4M^2(\not{x}\gamma_5 C)_{\alpha\beta}\Lambda_{\gamma} + \mathcal{A}_5M^2(\gamma_{\mu}\gamma_5 C)_{\alpha\beta}(i\sigma^{\mu\nu}x_{\nu}\Lambda)_{\gamma} + \mathcal{A}_6M^3(\not{x}\gamma_5 C)_{\alpha\beta}(\not{x}\Lambda)_{\gamma}, \quad (1)$$

where Λ_{γ} designates the spinor for the Λ baryon with momentum P . Since only axial-vector DAs contribute to the final sum rules of the process, we merely present this kind of structures for simplicity. The twist classification of those calligraphic DAs is indefinite, but they can be expressed by the ones with definite twist as

$$\begin{aligned} \mathcal{A}_1 &= A_1, & 2P \cdot x \mathcal{A}_2 &= -A_1 + A_2 - A_3, \\ 2\mathcal{A}_3 &= A_3, & 4P \cdot x \mathcal{A}_4 &= -2A_1 - A_3 - A_4 + 2A_5, \\ 4P \cdot x \mathcal{A}_5 &= A_3 - A_4, & (2P \cdot x)^2 \mathcal{A}_6 &= A_1 - A_2 + A_3 + A_4 - A_5 + A_6. \end{aligned} \quad (2)$$

The twist classification of A_i is given in Table. I. Each distribution amplitudes $F = A_i$ can be represented as Fourier

Twist-3	Twist-4	Twist-5	Twist-6
A_1	A_2, A_3	A_4, A_5	A_6

TABLE I: The twist for A_i .

integral over the longitudinal momentum fractions x_1, x_2, x_3 carried by the quarks inside the baryon with $\sum_i x_i = 1$,

$$F(a_i P \cdot x) = \int \mathcal{D}x e^{-ip \cdot x \sum_i x_i a_i} F(x_i).$$

The integration measure is defined as

$$\int \mathcal{D}x = \int_0^1 dx_1 dx_2 dx_3 \delta(x_1 + x_2 + x_3 - 1).$$

As elucidated in Ref. [23], those distribution amplitudes are scale dependent and can be expanded into orthogonal functions with increasing conformal spin. To the leading conformal spin, or s-wave, accuracy the explicit expansions read [13, 22]

$$\begin{aligned} A_1(x_i, \mu) &= -120x_1x_2x_3\phi_3^0(\mu), \\ A_2(x_i, \mu) &= -24x_1x_2\phi_4^0(\mu), \\ A_3(x_i, \mu) &= -12x_3(1-x_3)\psi_4^0(\mu), \\ A_4(x_i, \mu) &= -3(1-x_3)\psi_5^0(\mu), \\ A_5(x_i, \mu) &= -6x_3\phi_5^0(\mu), \\ A_6(x_i, \mu) &= -2\phi_6^0(\mu), \end{aligned} \quad (3)$$

where the constraint $A(x_1, x_2, x_3) = A(x_2, x_1, x_3)$ has been used in the derivation, which arises from the fact that the Λ baryon has isospin 0. All the six parameters involved in Eq. (3) can be expressed in terms of two independent matrix elements of local operators. Those parameters are expressed as

$$\phi_3^0 = \phi_6^0 = -f_{\Lambda}, \quad \phi_4^0 = \phi_5^0 = -\frac{1}{2}(\lambda_1 + f_{\Lambda}), \quad \psi_4^0 = \psi_5^0 = -\frac{1}{2}(\lambda_1 - f_{\Lambda}).$$

The nonperturbative parameter f_Λ originates from the following local matrix element:

$$\langle 0 | \epsilon_{ijk} [u^i(0) C \gamma_5 \not{z} d^j(0)] \not{z} s^k(0) | P \rangle = f_\Lambda z \cdot P \not{z} \Lambda(P). \quad (4)$$

The remaining parameter λ_1 is defined by the matrix element

$$\langle 0 | \epsilon_{ijk} [u^i(0) C \gamma_5 \gamma_\mu d^j(0)] \gamma^\mu s^k(0) | P \rangle = \lambda_1 M \Lambda(P). \quad (5)$$

III. $\Lambda_c \rightarrow \Lambda \ell^+ \nu$ DECAY FORM FACTORS FROM LIGHT-CONE SUM RULES

A. LCSR with CZ-type current

In compliance with the standard philosophy of the LCSR method, we start from the analysis of the following correlation function

$$z^\nu T_\nu(P, q) = i z^\nu \int d^4 x e^{iq \cdot x} \langle 0 | T \{ j_{\Lambda_c}(0) j_\nu(x) \} | P \rangle, \quad (6)$$

where $j_{\Lambda_c} = \epsilon_{ijk} (u^i C \gamma_5 \not{z} d^j) \not{z} c^k$ is the current interpolating the Λ_c baryon state which is similar to that used for $J^P = \frac{1}{2}^+$ octet baryons by Chernyak, Ogloblin and Zhitnitsky (CZ-type current) [25], $j_\nu = \bar{c} \gamma_\nu (1 - \gamma_5) s$ is the weak current, C is the charge conjugation matrix, and i, j, k denote the color indices. The auxiliary light-cone vector z is introduced to project out the main contribution onto the light cone. The coupling constant of the baryonic current to the vacuum can thus be defined as

$$\langle 0 | j_{\Lambda_c} | \Lambda_c(P') \rangle = f_{\Lambda_c} z \cdot P' \not{z} \Lambda_c(P'), \quad (7)$$

where $\Lambda_c(P')$ and P' is the Λ_c baryon spinor and four-momentum, respectively. Form factors are defined in the usual way

$$\begin{aligned} \langle \Lambda_c(P - q) | j_\nu | \Lambda(P) \rangle &= \bar{\Lambda}_c(P - q) \left[f_1 \gamma_\nu - i \frac{f_2}{M_{\Lambda_c}} \sigma_{\nu\mu} q^\mu \right. \\ &\quad \left. - \left(g_1 \gamma_\nu + i \frac{g_2}{M_{\Lambda_c}} \sigma_{\nu\mu} q^\mu \right) \gamma_5 \right] \Lambda(P), \end{aligned} \quad (8)$$

in which M_{Λ_c} is the Λ_c mass, $\Lambda(P)$ denotes the Λ spinor and satisfies $\not{P} \Lambda(P) = M \Lambda(P)$ with M the Λ mass and P its four-momentum. The form factors that give no contribution in the case of massless final leptons are omitted here.

With those definitions (7) and (8), the hadronic representation of the correlation function (6) can be written as

$$z^\nu T_\nu = \frac{2f_{\Lambda_c}}{M_{\Lambda_c}^2 - P'^2} (z \cdot P')^2 \left[f_1 \not{z} + f_2 \frac{\not{z} \not{q}}{M_{\Lambda_c}} - \left(g_1 \not{z} - g_2 \frac{\not{z} \not{q}}{M_{\Lambda_c}} \right) \gamma_5 \right] \Lambda(P) + \dots, \quad (9)$$

where $P' = P - q$ and the dots stand for the higher resonances and continuum contributions. While on the theoretical side, at large Euclidean momenta P'^2 and q^2 the correlation function (6) can be calculated perturbatively to the leading order of the QCD coupling α_s :

$$z^\nu T_\nu = -2(C \gamma_5 \not{z})_{\alpha\beta} (\not{z}(1 - \gamma_5))_\mu \int d^4 x \int \frac{d^4 k}{(2\pi)^4} \frac{z \cdot k}{k^2 - m_c^2} e^{i(k+q) \cdot x} \langle 0 | \epsilon_{ijk} u_\alpha^i(0) d_\beta^j(0) s_\mu^k(x) | P \rangle, \quad (10)$$

where m_c is the c -quark mass. Substituting (1) into Eq. (10) we obtain

$$\begin{aligned} z^\nu T_\nu &= -2(z \cdot P)^2 \left[\int dx_3 \frac{x_3 B_0(x_3)}{k^2 - m_c^2} + M^2 \int dx_3 \frac{x_3^2 B_1(x_3)}{(k^2 - m_c^2)^2} + 2M^4 \int dx_3 \frac{x_3^3 B_2(x_3)}{(k^2 - m_c^2)^3} \right] \not{z}(1 - \gamma_5) \Lambda(P) \\ &\quad + 2(z \cdot P)^2 \left[M \int dx_3 \frac{x_3 B_3(x_3)}{(k^2 - m_c^2)^2} + 2M^3 \int dx_3 \frac{x_3^2 B_2(x_3)}{(k^2 - m_c^2)^3} \right] \not{z} \not{q}(1 + \gamma_5) \Lambda(P) + \dots, \end{aligned} \quad (11)$$

where $k = x_3 P - q$ and the ellipses stand for contributions that are nonleading in the infinite momentum frame kinematics $P \rightarrow \infty$, $q \sim \text{const.}$, $z \sim 1/P$. The functions B_i are defined by

$$\begin{aligned} B_0(\alpha_3) &= \int_0^{1-x_3} dx_1 A_1(x_1, 1-x_1-x_3, x_3), \\ B_1(\alpha_3) &= -2\tilde{A}_1 + \tilde{A}_2 - \tilde{A}_3 - \tilde{A}_4 + \tilde{A}_5, \\ B_2(\alpha_3) &= \tilde{\tilde{A}}_1 - \tilde{\tilde{A}}_2 + \tilde{\tilde{A}}_3 + \tilde{\tilde{A}}_4 - \tilde{\tilde{A}}_5 + \tilde{\tilde{A}}_6, \\ B_3(\alpha_3) &= -\tilde{A}_1 + \tilde{A}_2 - \tilde{A}_3. \end{aligned} \quad (12)$$

The DAs with tildes are defined via integration as follows

$$\begin{aligned} \tilde{A}(x_3) &= \int_1^{x_3} dx'_3 \int_0^{1-x'_3} dx_1 A(x_1, 1-x_1-x'_3, x'_3), \\ \tilde{\tilde{A}}(x_3) &= \int_1^{x_3} dx'_3 \int_1^{x'_3} dx''_3 \int_0^{1-x''_3} dx_1 A(x_1, 1-x_1-x''_3, x''_3). \end{aligned} \quad (13)$$

These functions originate from the partial integration, which is used to eliminate the factors $1/(P \cdot x)^n$ appearing in the distribution amplitudes. When the next-to-leading order conformal expansion is considered, the surface terms completely sum to zero. The term B_0 corresponds to the leading twist contribution. The form factors f_2 and g_2 in Eq. (11) are characterized by the higher twist contributions.

Matching Eqs. (9) and (11), adopting the quark-hadron duality assumption and employing a Borel improvement of P'^2 on both sides lead us to the desired sum rules for the form factors f_1 and f_2 :

$$\begin{aligned} -f_{\Lambda_c} f_1 e^{-M_{\Lambda_c}^2/M_B^2} &= -\int_{x_0}^1 dx_2 e^{-s'/M_B^2} \left[B_0 + \frac{M^2}{M_B^2} \left(-B_1(x_3) + \frac{M^2}{M_B^2} B_2(x_3) \right) \right] \\ &\quad + \frac{M^2 x_0^2 e^{-s_0/M_B^2}}{m_c^2 - q^2 + x_0^2 M^2} \left(B_1(x_0) - \frac{M^2}{M_B^2} B_2(x_0) \right) \\ &\quad + \frac{M^2 e^{-s_0/M_B^2} x_0^2}{m_c^2 - q^2 + x_0^2 M^2} \frac{d}{dx_0} \left(\frac{M^2 x_0^2 B_2(x_0)}{m_c^2 - q^2 + x_0^2 M^2} \right), \end{aligned} \quad (14a)$$

$$\begin{aligned} \frac{f_{\Lambda_c} f_2}{M_{\Lambda_c} M} e^{-M_{\Lambda_c}^2/M_B^2} &= \frac{1}{M_B^2} \int_{x_0}^1 \frac{dx_3}{x_3} e^{-s'/M_B^2} \left(B_3(x_3) - \frac{M^2}{M_B^2} B_2(x_3) \right) \\ &\quad + \frac{x_0 e^{-s_0/M_B^2}}{m_c^2 - q^2 + x_0^2 M^2} \left(B_3(x_0) - \frac{M^2}{M_B^2} B_2(x_0) \right) \\ &\quad + \frac{M^2 e^{-s_0/M_B^2} x_0^2}{m_c^2 - q^2 + x_0^2 M^2} \frac{d}{dx_0} \left(\frac{x_0 B_2(x_0)}{m_c^2 - q^2 + x_0^2 M^2} \right), \end{aligned} \quad (14b)$$

where

$$s' = (1-x)M^2 + \frac{m_c^2 - (1-x)q^2}{x},$$

and x_0 is the positive solution of the quadratic equation for $s' = s_0$:

$$2M^2 x_0 = \sqrt{(-q^2 + s_0 - M^2)^2 + 4M^2(-q^2 + m_c^2)} - (-q^2 + s_0 - M^2).$$

As the sum rules for the form factors g_1 and g_2 are identical with those for the f_1 and f_2 , $f_1 = g_1$ and $f_2 = g_2$, we will only discuss the results for f_1 and f_2 in the numerical analysis section.

B. LCSR with Ioffe-type interpolating current

The interpolating current used for the baryon is not the unique one. As exemplified in the studies [24] for the applications of QCD sum rules, there are other choices existing in interpolating baryonic state with the same quantum

numbers. It has been known that the current interpolating the hadron state is of great importance in field theory and can affect the calculations to some extent [14, 26, 27]. In this subsection, we adopt another interpolating current to investigate the form factors of the semi-leptonic decay $\Lambda_c \rightarrow \Lambda \ell^+ \nu$. To this end, the Ioffe-type baryonic current (see [24] for a reference) is used in the calculation, that is $j_{\Lambda_c} = \epsilon_{ijk}(u^i C \gamma_5 \gamma_\mu d^j) \gamma^\mu c^k$. The coupling of this kind of interpolating current is determined by the following matrix element:

$$\langle 0 | j_{\Lambda_c} | \Lambda_c(P') \rangle = \lambda_{1c} M_{\Lambda_c} \Lambda_c(P'). \quad (15)$$

In order to derive the LCSR of the weak transition form factors, we need to express the correlation function (6) both phenomenologically and theoretically. By inserting a complete set of intermediate states with the same quantum numbers as those of Λ_c , the hadronic representation of the correlation function is expressed as

$$\begin{aligned} z^\nu T_\nu = & \frac{\lambda_{1c} M_{\Lambda_c}}{M_{\Lambda_c}^2 - P'^2} \left[2P \cdot z f_1(q^2) + \frac{2P \cdot z}{M_{\Lambda_c}} \not{q}_\perp f_2(q^2) - [M_\Lambda f_1(q^2) - \frac{q^2}{M_{\Lambda_c}} f_2(q^2) - M_{\Lambda_c} f_1(q^2)] \not{z} \right. \\ & + [f_1(q^2) + f_2(q^2) + \frac{M_\Lambda}{M_{\Lambda_c}} f_2(q^2)] \not{z} \not{q} - 2P \cdot z g_1(q^2) \gamma_5 + \frac{2P \cdot z}{M_{\Lambda_c}} g_2(q^2) \not{q}_\perp \gamma_5 \\ & \left. - [M_\Lambda g_1(q^2) - \frac{q^2}{M_{\Lambda_c}} g_2(q^2) + M_{\Lambda_c} g_1(q^2)] \not{z} \gamma_5 - [g_1(q^2) - g_2(q^2) + \frac{M_\Lambda}{M_{\Lambda_c}} g_2(q^2)] \not{z} \not{q} \gamma_5 \right] \Lambda(P) + \dots, \quad (16) \end{aligned}$$

where $\not{q}_\perp = \not{q} - \frac{p \cdot q}{p \cdot z} \not{z}$, and the dots stand for the higher resonance and continuum contributions. In contrast to the case of CZ-type current, there are more Lorentz structures existing in the expression. Generally speaking, all of the Lorentz structures can give information on the form factors. In this paper we choose the terms proportional to 1 and

\not{q}_\perp to get the sum rules. After the standard procedure of LCSR method, we arrive at the following sum rules:

$$\begin{aligned}
\lambda_{1c} M_{\Lambda_c} f_1(q^2) = & \int_{\alpha_{30}}^1 d\alpha_3 e^{-\frac{s-M_{\Lambda_c}^2}{M_B^2}} \left\{ -\frac{m_c}{\alpha_3} B_0(\alpha_3) + M B'_0(\alpha_3) + \frac{M}{\alpha_3 M_B^2} (M_B^2 + \frac{q^2}{\alpha_3} + M m_c) B_3(\alpha_3) \right. \\
& + \frac{M^2 m_c}{\alpha_3 M_B^2} B_5(\alpha_3) + \frac{M^2}{\alpha_3 M_B^2} (\alpha_3 M + m_c) B_4(\alpha_3) - \frac{M^4 m_c}{\alpha_3 M_B^4} B_2(\alpha_3) \Big\} \\
& + e^{-\frac{s_0-M_{\Lambda_c}^2}{M_B^2}} \frac{1}{\alpha_{30}^2 M^2 + m_c^2 - q^2} \left\{ (M q^2 + \alpha_{30} M^2 m_c) B_3(\alpha_{30}) + \alpha_{30} M^2 m_c B_5(\alpha_{30}) \right. \\
& + M^2 (\alpha_{30}^2 M + \alpha_{30} m_c) B_4(\alpha_{30}) - \frac{\alpha_{30} M^4 m_c}{M_B^2} B_2(\alpha_{30}) \Big\} \\
& + e^{-\frac{s_0-M_{\Lambda_c}^2}{M_B^2}} \frac{\alpha_{30}^2}{\alpha_{30}^2 M^2 + m_c^2 - q^2} \frac{d}{d\alpha_{30}} \frac{\alpha_{30} M^4 m_c}{\alpha_{30}^2 M^2 + m_c^2 - q^2} B_2(\alpha_{30}), \tag{17a}
\end{aligned}$$

$$\begin{aligned}
\lambda_{1c} f_2(q^2) = & \int_{\alpha_{30}}^1 d\alpha_3 e^{-\frac{s-M_{\Lambda_c}^2}{M_B^2}} \left\{ \frac{1}{\alpha_3} B_0(\alpha_3) - \frac{M}{\alpha_3 M_B^2} (M + \frac{m_c}{\alpha_3}) B_3(\alpha_3) - \frac{M^2}{\alpha_3 M_B^2} B_4(\alpha_3) + \frac{M^3 m_c}{\alpha_3^2 M_B^4} B_2(\alpha_3) \right\} \\
& - e^{-\frac{s_0-M_{\Lambda_c}^2}{M_B^2}} \frac{1}{\alpha_{30}^2 M^2 + m_c^2 - q^2} \left\{ (\alpha_{30} M^2 + M m_c) B_3(\alpha_{30}) + \alpha_{30} M^2 B_4(\alpha_{30}) - \frac{M^3 m_c}{M_B^2} B_2(\alpha_{30}) \right\} \\
& - e^{-\frac{s_0-M_{\Lambda_c}^2}{M_B^2}} \frac{\alpha_{30}^2}{\alpha_{30}^2 M^2 + m_c^2 - q^2} \frac{d}{d\alpha_{30}} \frac{M^3 m_c}{\alpha_{30}^2 M^2 + m_c^2 - q^2} B_2(\alpha_{30}), \tag{17b}
\end{aligned}$$

$$\begin{aligned}
\lambda_{1c} M_{\Lambda_c} g_1(q^2) = & \int_{\alpha_{30}}^1 d\alpha_3 e^{-\frac{s-M_{\Lambda_c}^2}{M_B^2}} \left\{ -\frac{m_c}{\alpha_3} B_0(\alpha_3) - M B'_0(\alpha_3) - \frac{M}{\alpha_3 M_B^2} (\frac{q^2}{\alpha_3} + M_B^2 - M m_c) B_3(\alpha_3) \right. \\
& + \frac{M^2 m_c}{\alpha_3 M_B^2} B_5(\alpha_3) + \frac{M^2}{\alpha_3 M_B^2} (-\alpha_3 M + m_c) B_4(\alpha_3) - \frac{M^4 m_c}{\alpha_3 M_B^4} B_2(\alpha_3) \Big\} \\
& - e^{-\frac{s_0-M_{\Lambda_c}^2}{M_B^2}} \frac{1}{\alpha_{30}^2 M^2 + m_c^2 - q^2} \left\{ (M q^2 - \alpha_{30} M^2 m_c) B_3(\alpha_{30}) - \alpha_{30} M^2 m_c B_5(\alpha_{30}) \right. \\
& + M^2 (\alpha_{30}^2 M - \alpha_{30} m_c) B_4(\alpha_{30}) + \frac{\alpha_{30} M^4 m_c}{M_B^2} B_2(\alpha_{30}) \Big\} \\
& + e^{-\frac{s_0-M_{\Lambda_c}^2}{M_B^2}} \frac{\alpha_{30}^2}{\alpha_{30}^2 M^2 + m_c^2 - q^2} \frac{d}{d\alpha_{30}} \frac{\alpha_{30} M^4 m_c}{\alpha_{30}^2 M^2 + m_c^2 - q^2} B_2(\alpha_{30}), \tag{17c}
\end{aligned}$$

$$\begin{aligned}
\lambda_{1c} g_2(q^2) = & \int_{\alpha_{30}}^1 d\alpha_3 e^{-\frac{s-M_{\Lambda_c}^2}{M_B^2}} \left\{ -\frac{1}{\alpha_3} B_0(\alpha_3) + \frac{M}{\alpha_3 M_B^2} (M - \frac{m_c}{\alpha_3}) B_3(\alpha_3) + \frac{M^2}{\alpha_3 M_B^2} B_4(\alpha_3) + \frac{M^3 m_c}{\alpha_3^2 M_B^4} B_2(\alpha_3) \right\} \\
& + e^{-\frac{s_0-M_{\Lambda_c}^2}{M_B^2}} \frac{1}{\alpha_{30}^2 M^2 + m_c^2 - q^2} \left\{ (\alpha_{30} M^2 - M m_c) B_3(\alpha_{30}) + \alpha_{30} M^2 B_4(\alpha_{30}) + \frac{M^3 m_c}{M_B^2} B_2(\alpha_{30}) \right\} \\
& - e^{-\frac{s_0-M_{\Lambda_c}^2}{M_B^2}} \frac{\alpha_{30}^2}{\alpha_{30}^2 M^2 + m_c^2 - q^2} \frac{d}{d\alpha_{30}} \frac{M^3 m_c}{\alpha_{30}^2 M^2 + m_c^2 - q^2} B_2(\alpha_{30}), \tag{17d}
\end{aligned}$$

where the additional definitions are used as

$$\begin{aligned}
B'_0(\alpha_3) &= \int_0^{1-x_3} dx_1 A_3(x_1, 1-x_1-x_3, x_3), \\
B_4(\alpha_3) &= \tilde{A}_3 - \tilde{A}_4, \\
B_5(\alpha_3) &= -\tilde{A}_1 - \tilde{A}_3 + \tilde{A}_5. \tag{18}
\end{aligned}$$

IV. NUMERICAL ANALYSIS AND THE CONCLUSION

A. Values for f_Λ and λ_1

The explicit expressions of the baryon DAs rely on the nonperturbative parameters f_Λ and λ_1 , which need to be determined by some nonperturbative method. In the present subsection we use the QCD sum rule approach to estimate the values. According to their definitions, we consider correlation functions

$$\Pi(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T \{ J_i(x) \bar{J}_j(0) \} | 0 \rangle, \quad (19)$$

where J_i are currents given in (4) and (5). Following the standard procedure, the sum rules of f_Λ, λ_1 and their relative sign are straightforward

$$(4\pi)^4 f_\Lambda^2 e^{-M^2/M_B^2} = \frac{2}{5} \int_{m_s^2}^{s_0} s(1-x)^5 e^{-s/M_B^2} ds - \frac{b}{3} \int_{m_s^2}^{s_0} x(1-x)(1-2x) e^{-s/M_B^2} \frac{ds}{s}, \quad (20)$$

$$\begin{aligned} 4(2\pi)^4 \lambda_1^2 M^2 e^{-M^2/M_B^2} &= \frac{1}{2} \int_{m_s^2}^{s_0} s^2 [(1-x)(1+x)(1-8x+x^2) - 12x^2 \ln x] e^{-s/M_B^2} ds \\ &+ \frac{b}{12} \int_{m_s^2}^{s_0} (1-x)^2 e^{-s/M_B^2} ds - \frac{4}{3} a^2 \left(1 - \frac{m_0^2}{2M_B^2} - \frac{m_0^2 m_s^2}{2M_B^4}\right. \\ &\left. + \frac{m_0^4 m_s^4}{16M_B^8}\right) e^{-m_s^2/M_B^2} - m_s a_s \int_{m_s^2}^{s_0} e^{-s/M_B^2} ds, \end{aligned} \quad (21)$$

$$\begin{aligned} (4\pi)^4 f_\Lambda \lambda_1^* M e^{-M^2/M_B^2} &= \frac{2}{3} m_s \int_{m_s^2}^{s_0} s [(1-x)(3+13x-5x^2+x^3) + 12x \ln x] e^{-s/M_B^2} ds \\ &+ \frac{b}{3} m_s \int_{m_s^2}^{s_0} \frac{1}{s} (1-x) \left[1 + \frac{(1-x)(2-5x)}{3x}\right] e^{-s/M_B^2} ds + \frac{8}{3} a_s \int_{m_s^2}^{s_0} e^{-s/M_B^2} ds. \end{aligned} \quad (22)$$

where $x = m_s^2/s$ and m_s is the s -quark mass. The sum rule for the f_Λ has been obtained before [11] where the corresponding heavy quark limit is also derived. At the working window $s_0 \sim 2.55 \text{ GeV}^2$ and $1 < M_B^2 < 2 \text{ GeV}^2$ the numerical values for the coupling constants read

$$f_\Lambda = 6.0 \times 10^{-3} \text{ GeV}^2, \quad \lambda_1 = 1.0 \times 10^{-2} \text{ GeV}^2. \quad (23)$$

The relative sign of λ_1 and f_Λ is obtained from the sum rule (22). In the numeric analysis, the standard values $a = -(2\pi)^2 \langle \bar{q}q \rangle = 0.55 \text{ GeV}^3$, $a_s = -(2\pi)^2 \langle \bar{s}s \rangle = 0.8a$, $b = (2\pi)^2 \langle \alpha_s G^2/\pi \rangle = 0.47 \text{ GeV}^4$ and $m_s = 0.15 \text{ GeV}$ are adopted. It should be noted that our value for f_Λ here does coincide with that obtained in Ref. [25]. In comparison with the previous work [15], sum rules of λ_1^2 and $f_\Lambda \lambda_1$ are renewed with the consideration of four-quark condensate contributions a^2 . The numerical analysis shows that the renewed sum rules give a different sign to the parameter λ_1 , which may lead to the correct results on the electromagnetic form factors [14].

The sum rules for the coupling of Λ_c to vacuum, f_{Λ_c} and λ_{1c} , are similar to that for Λ , where the simple substitution $m_s \rightarrow m_c$ should be made. The numerical estimates are $f_{\Lambda_c} = (9.1 \pm 0.5) \times 10^{-3} \text{ GeV}^2$ and $\lambda_{1c} = (1.2 \pm 0.2) \times 10^{-2} \text{ GeV}^2$, taken from the interval $2 < M_B^2 < 3 \text{ GeV}^2$ for f_{Λ_c} with $s_0 \sim 10 \text{ GeV}^2$ and $1.5 < M_B^2 < 2.0 \text{ GeV}^2$ for λ_{1c} with $s_0 \sim 9 \text{ GeV}^2$.

B. Analysis of the LCSRs

In the numerical analysis for the sum rules of the form factors, the charm quark mass is taken to be $m_c = 1.27 \text{ GeV}$, and the other relevant parameters, Λ_c and Λ baryon masses and the value of $|V_{cs}|$, are used as the centra values

provided by PDG [28]. The analysis starts from the sum rules with the CZ-type interpolating current. We first take into account contributions from the twist-3 DAs, in which only twist-3 DA is kept. Substituting the above given parameters into the LCSRs and varying the continuum threshold within the range $s_0 = 9 - 11 \text{ GeV}^2$, we find there exist an acceptable stability in the working window $M_B^2 = 8 - 10 \text{ GeV}^2$ for the Borel parameter. The M_B^2 and the q^2 dependence for the corresponding form factors are shown in Figs. 1 and 2, respectively. Also given in Fig. 2 are two leading twist results, corresponding to contributions only retaining B_0 in the sum rules. It is apparent that only f_1 survives in the approximation. Apart from the leading twist DA we discuss in Eq. (3), there still exists another form from Chernyak, Ogloblin and Zhitnitsky [25]:

$$A_1^{COZ}(x_i) = -21\varphi_{as}[0.52(x_1^2 + x_2^2) + 0.68x_3^2 - 1.12x_1x_2 - 0.48x_3(x_1 + x_2)], \quad (24)$$

where $\varphi_{as} = 120x_1x_2x_3$ is the asymptotic DA. The corresponding result is also illustrated for contrast.

When including contributions up to twist-6, the stability is agreeable within the range $s_0 = 9 - 11 \text{ GeV}^2$ and $M_B^2 = 7 - 9 \text{ GeV}^2$. In that working region, the twist-3 contribution to f_1 is the dominant one, while for f_2 , the main contribution comes from the twist-4 DAs and its magnitude is approximately ~ 1.5 of the twist-3 one in the whole dynamical region, but with a different sign. On account of the relatively small momentum transfer, the asymptotic behavior of DAs may not be fulfilled and we need incorporate higher conformal spin in the expansion for them. Furthermore, QCD sum rule tends to overestimate the higher conformal spin expansion parameters [29], and the corresponding parameter will enter in the coefficients of the higher conformal spin expansion, which is well known as the Wandzura-Wilczek type contribution. The M_B^2 and the q^2 dependence for the corresponding form factors are also shown in Figs. 1 and 2.

Both form factors can be well fitted by the three-parameter dipole formula in the LCSR allowed region, $0 < q^2 < 0.8 \text{ GeV}^2$:

$$f_i(q^2) = \frac{f_i(0)}{a_2(q^2/M_{\Lambda_c}^2)^2 + a_1q^2/M_{\Lambda_c}^2 + 1}. \quad (25)$$

Below in Table II we give the central values of those coefficients with parameters $M_B^2 = 9 \text{ GeV}^2$ and $s_0 = 10 \text{ GeV}^2$. It is noted that in the table, $f_1 = g_1$ and $f_2 = g_2$ for the CZ-type interpolating current.

For the analysis of sum rules from the Ioffe-type interpolating current, we comply with the same procedure above. The calculation shows that the form factors vary mildly with the Borel parameter in the range $8 \text{ GeV}^2 \leq M_B^2 \leq 10 \text{ GeV}^2$ with the threshold varying in the region $8 \text{ GeV}^2 \leq s_0 \leq 10 \text{ GeV}^2$. Therefore in the following analysis we set $M_B^2 = 9 \text{ GeV}^2$. The q^2 -dependence of the weak transition form factors are plotted in Fig. 3, with both contributions from twist-3 and up to twist-6. The figure shows that higher order twist contributions can play so important roles in the calculation that it is necessary to include them in the investigations. All the form factors f_i and g_i can be well

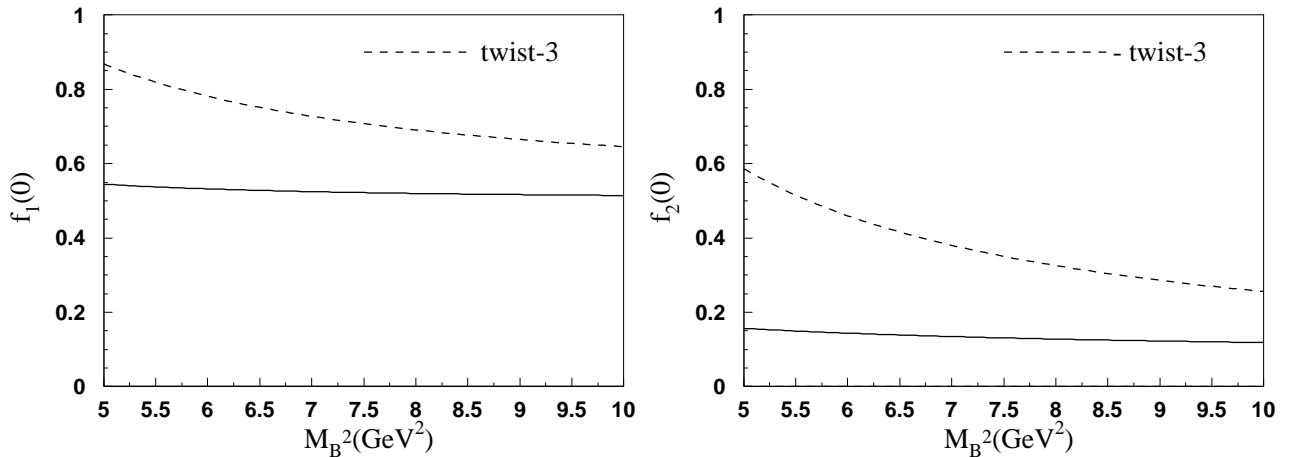


FIG. 1: The dependence on M_B^2 of the LCSRs for the form factors f_1 and f_2 at $q^2 = 0$. The continuum threshold is $s_0 = 10 \text{ GeV}^2$.

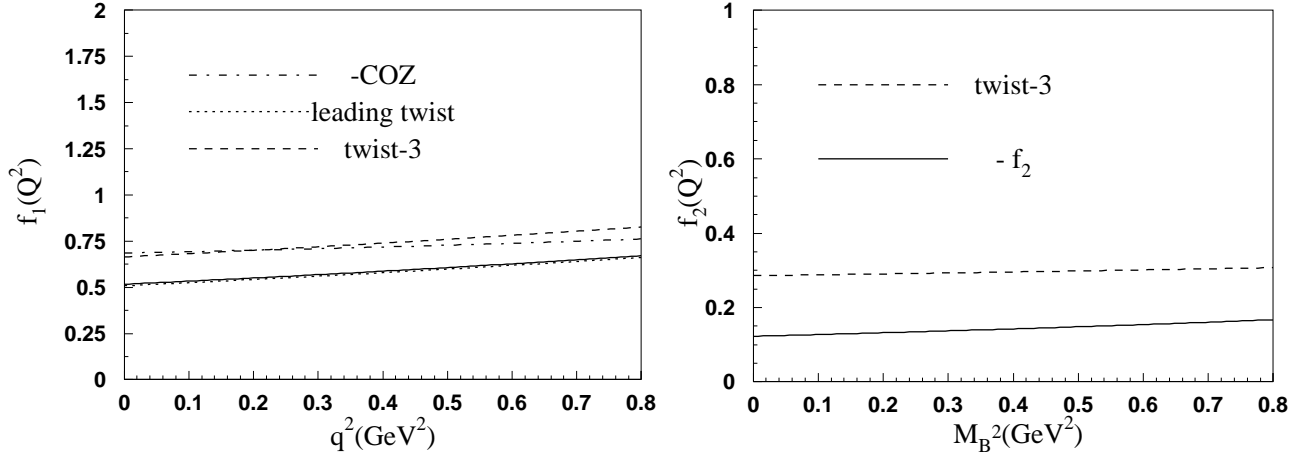


FIG. 2: The dependence on q^2 of the LCSRs for the form factors f_1 and f_2 . The “COZ” denotes the result obtained from the COZ DAs. The continuum threshold and the Borel parameter are $s_0 = 10\text{GeV}^2$ and $M_B^2 = 9\text{GeV}^2$.

	CZ-type						Ioffe-type					
	Twist-3			Twist-6			Twist-3			Twist-6		
	$f_i(0)$	a_1	a_2	$f_i(0)$	a_1	a_2	$f_i(0)$	a_1	a_2	$f_i(0)$	a_1	a_2
f_1	0.665	-1.350	0.460	0.517	-1.692	0.941	-0.991	-2.361	1.521	-0.387	-1.920	1.915
f_2	-0.285	-0.421	-0.243	0.123	-1.902	1.080	1.309	-2.066	-0.0001	0.664	-2.251	0.648
g_1	0.665	-1.350	0.460	0.517	-1.692	0.941	-0.339	-2.481	0.101	-0.702	-3.219	2.763
g_2	-0.285	-0.421	-0.243	0.123	-1.902	1.080	-0.847	-3.357	4.020	-1.208	-3.178	3.114

TABLE II: The dipole formula fit for the form factors f_i and g_i with two kinds of interpolating currents.

fitted by the dipole formula (25) in the range $0 < q^2 < 0.8\text{GeV}^2$ with $s_0 = 9\text{GeV}^2$. The fit coefficients are shown in Table II.

C. Semileptonic decay width

The differential decay rate of the decay process can be expressed by the weak transition form factors as

$$\begin{aligned}
\frac{d\Gamma}{dq^2} = & \frac{G_F^2 |V_{cs}|^2}{192\pi^3 M_{\Lambda_c}^5} q^2 \sqrt{q_+^2 q_-^2} \left\{ -6f_1 f_2 M_{\Lambda_c} m_+ q_-^2 + 6g_1 g_2 M_{\Lambda_c} m_- q_+^2 \right. \\
& + f_1^2 M_{\Lambda_c}^2 \left(\frac{m_+^2 m_-^2}{q^2} + m_-^2 - 2(q^2 + 2M_{\Lambda_c} M_{\Lambda}) \right) \\
& + g_1^2 M_{\Lambda_c}^2 \left(\frac{m_+^2 m_-^2}{q^2} + m_+^2 - 2(q^2 - 2M_{\Lambda_c} M_{\Lambda}) \right) \\
& - f_2^2 [-2m_+^2 m_-^2 + m_+^2 q^2 + q^2 (q^2 + 4M_{\Lambda_c} M_{\Lambda})] \\
& \left. - g_2^2 [-2m_+^2 m_-^2 + m_-^2 q^2 + q^2 (q^2 - 4M_{\Lambda_c} M_{\Lambda})] \right\}, \tag{26}
\end{aligned}$$

where $m_{\pm} = M_{\Lambda_c} \pm M_{\Lambda}$ and $q_{\pm} = q^2 - m_{\pm}^2$ are used for convenience. It has been known that the light-cone sum rules on the decay form factors can only be reliable in the range $q^2 - m_c^2 \ll 0$, i.e. $0 < q^2 < 0.8\text{GeV}^2$ in our calculations, while the thorough understanding of the decay process needs us to know information on the whole physical region $0 < q^2 < (M_{\Lambda_c} - M_{\Lambda})^2$. In order to give numerical estimate, we extrapolate the fit formula to the whole physical region, assuming that the form factors can be described by the dipole formula with the same coefficients in the whole kinematic range.

After extrapolation of the form factors given in Table II, we calculate the differential decay rate for the process $\Lambda_c \rightarrow \Lambda \ell^+ \nu$, which is shown in Fig. 4 (a) for CZ-type interpolating current and Fig. 4 (b) for Ioffe-type current. The estimate of the total decay width can be obtained after integration of the momentum transfer q^2 in the whole kinematical region. Taking into account contributions up to twist-6 DAs, we give the prediction $\Gamma(\Lambda_c \rightarrow \Lambda \ell^+ \nu) = (10.04 \pm 0.88) \times 10^{-14} \text{ GeV}$ for CZ-type current and $\Gamma(\Lambda_c \rightarrow \Lambda \ell^+ \nu) = (6.45 \pm 1.06) \times 10^{-14} \text{ GeV}$ for Ioffe-type current. The errors in our numerical results come from the different choice of the threshold $s_0 = 9 - 11 \text{ GeV}^2$ with the Borel parameter varying in the region $7 \text{ GeV}^2 \leq M_B^2 \leq 9 \text{ GeV}^2$ for CZ-type current and $s_0 = 8 - 10 \text{ GeV}^2$ with $8 \text{ GeV}^2 \leq M_B^2 \leq 10 \text{ GeV}^2$ for Ioffe-type current. Note that, as the estimations are from the dipole formula fits with the sum rules as the input data, the uncertainty due to the variation of the input parameters, such as f_Λ and λ_1 , as well as f_{Λ_c} and λ_{1c} , is not included, which may reach 5-10% or more.

For a comparison with experiments, we turn to data provided by PDG. By use of the mean lifetime of Λ_c : $\tau = 200 \times 10^{-15} \text{ s}$, the branching ratio of the process is estimated to be $Br(\Lambda_c \rightarrow \Lambda \ell^+ \nu) = 0.030 \pm 0.003$ for CZ-type interpolating current and $Br(\Lambda_c \rightarrow \Lambda \ell^+ \nu) = 0.020 \pm 0.003$ for Ioffe-type interpolating current, which are explicitly shown in Table III. In order to see influence of higher twist DAs to the decay, we also give predictions from the twist-3 contributions for the two kinds of interpolating currents in the table. The table shows that the adoption of Ioffe-type interpolating current is a better choice for the study of the semi-leptonic decay mode $\Lambda_c \rightarrow \Lambda \ell^+ \nu$, and the inclusion of higher twist contributions is necessary for calculations. It can also be seen that the improvement of the parameters in the distribution amplitudes results in marked difference from results in [15] for the CZ-type current case.

To summarize, we have given an approved investigation on the semi-leptonic decay $\Lambda_c \rightarrow \Lambda \ell^+ \nu$. The form factors

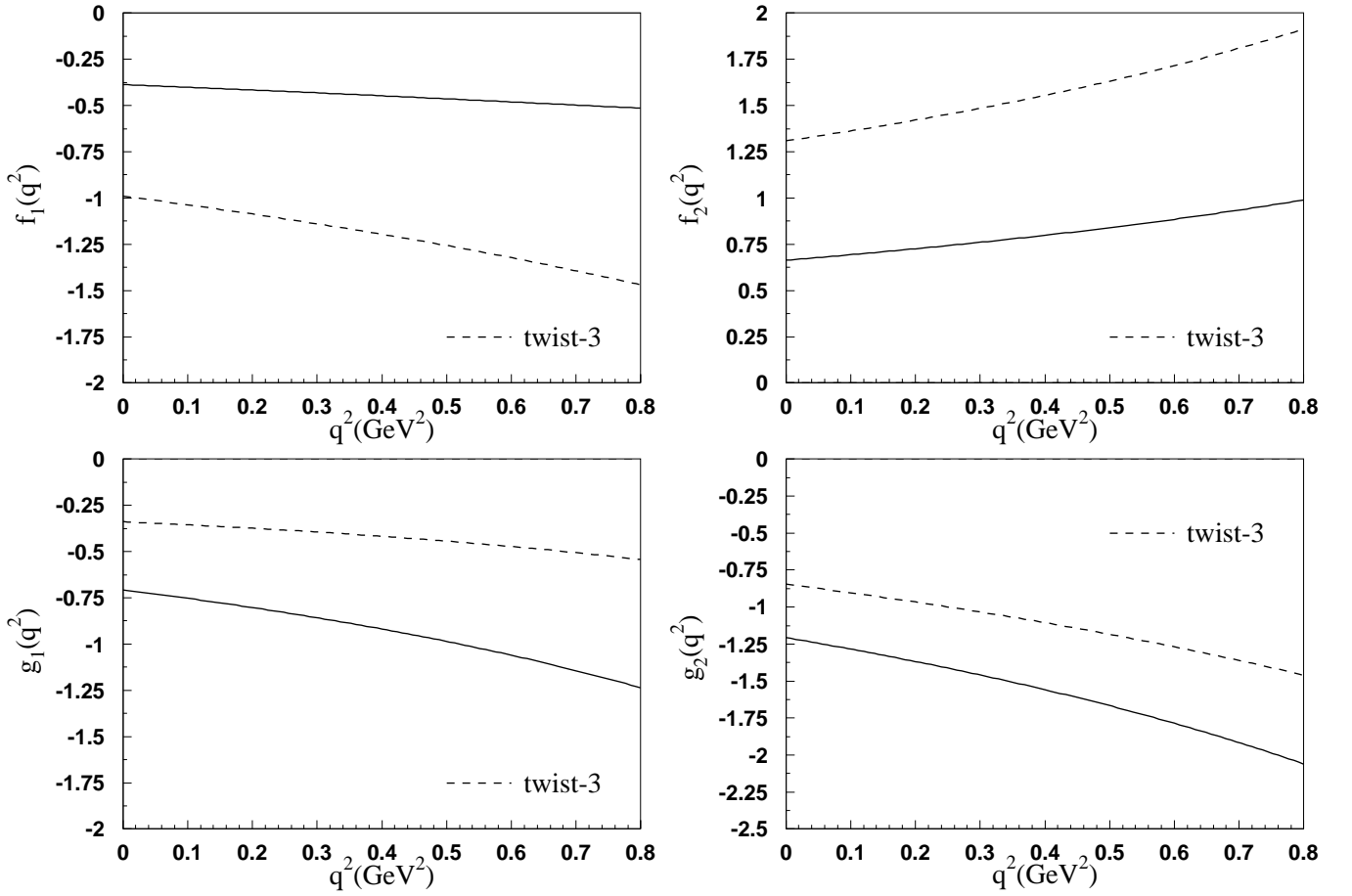


FIG. 3: The q^2 -dependence of the LCSRs for the form factors f_i and g_i with Ioffe-type interpolating current. The continuum threshold is $s_0 = 9 \text{ GeV}^2$ and the Borel parameter is set to be $M_B^2 = 9 \text{ GeV}^2$.

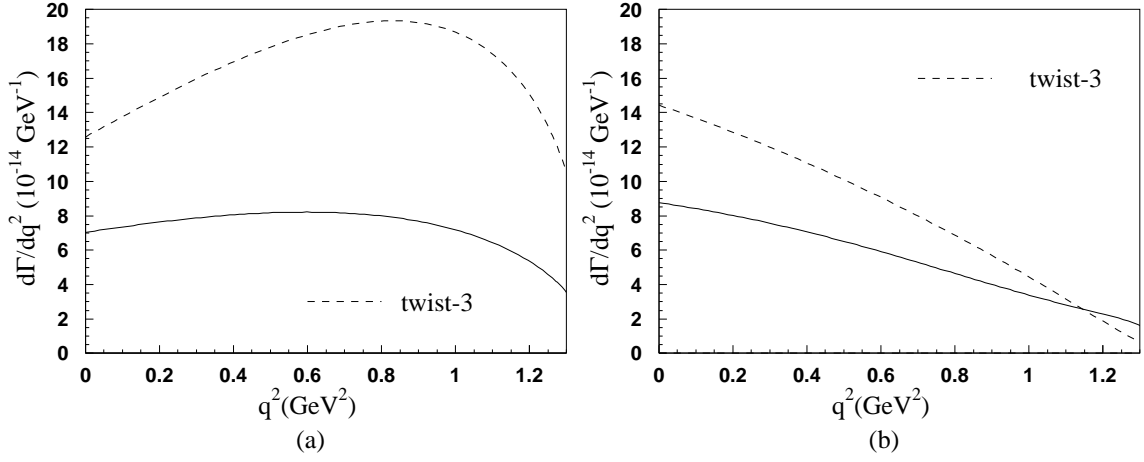


FIG. 4: Differential decay rate for $\Lambda_c \rightarrow \Lambda \ell^+ \nu$. (a) corresponds to the CZ-type interpolating current and (b) the Ioffe-type current.

	$\Gamma(\times 10^{-14})\text{GeV}$			$Br(\%)$		
	Twist-3	Twist-6	PDG	Twist-3	Twist-6	PDG
CZ-type	19.7	10.04	6.77	5.8	3.0	2.0
Ioffe-type	10.5	6.5		3.2	2.0	

TABLE III: The prediction of the semi-leptonic decay $\Lambda_c \rightarrow \Lambda \ell^+ \nu$ with two kinds of interpolating currents.

characterizing the process are studied within the framework of the LCSR method. In the calculations we adopt both CZ-type and Ioffe-type current as interpolating field for the Λ_c baryon. Light-cone sum rules for the form factors are derived and used to predict the decay width, which is $\Gamma(\Lambda_c \rightarrow \Lambda \ell^+ \nu) = (10.04 \pm 0.88) \times 10^{-14} \text{GeV}$ from CZ-type interpolating current and $\Gamma(\Lambda_c \rightarrow \Lambda \ell^+ \nu) = (6.45 \pm 1.06) \times 10^{-14} \text{GeV}$ from Ioffe-type interpolating current. The results show that the adoption of the Ioffe-type interpolating current is in better agreement with the experimental data. This is partly due to the fact that in the case when Ioffe-type interpolator is used, terms proportional to the mass of the heavy quark appearing in the sum rules (17) can play an important role in the numerical analysis. In addition, the analyses also show that the higher twist contributions are important for the results so that they need to be included in the calculation.

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- [1] I.I. Balitsky, V.M. Braun and A.V. Kolesnichenko, Nucl. Phys. B **312**, 509(1989); V.L. Chernyak and I.R. Zhitnitsky, *ibid.* B **345**, 137 (1990).
 - [2] V. Braun, Light-Cone Sum Rules, hep-ph/9801222; P. Colangelo and A. Khodjamirian, in *At the Frontier of Particle Physics/Handbook of QCD*, edited by M. Shifman (World Scientific, Singapore, 2001),p. 1495.
 - [3] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979); V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Fortschr. Phys. **32**, 11 (1984).
 - [4] P. Ball and V. M. Braun, Phys. Rev. D **55**, 5561 (1997).
 - [5] G. P. Lepage and S. J. Brodsky, Phys. Rev. Lett. **43**, 545(1979); **43**, 1625(E) (1979); G. P. Lepage and S. J. Brodsky, Phys. Rev. D. **22**, 2157 (1980); V. L. Chernyak and A. R. Zhitnitsky, Phys. Rept. **112**, 173 (1984).
 - [6] V. M. Braun, A. Lenz, N. Mahnke, and E. Stein, phys. Rev. D **65**, 074011 (2002);

- [7] A. Lenz, M. Wittmann and E. Stein, Phys. Lett. B **581**, 199 (2004).
- [8] V. M. Braun, A. Lenz and M. Wittmann, Phys. Rev. D **73**, 094019 (2006); A. Lenz, M. Göckeler, T. Kaltenbrunner and N. Warkentin, Phys. Rev. D **79**, 093007 (2009).
- [9] Z. G. Wang, S. L. Wan and W. M. Yang, Phys. Rev. D **73**, 094011 (2006); hep-ph/0601060.
- [10] T. M. Aliev, K. Azizi, and A. Ozpineci, M. Savci, Phys. Rev. D **77** (2008) 114014.
- [11] M.Q. Huang and D.W. Wang, Phys. Rev. D **69**, 094003 (2004).
- [12] V. M. Braun, A. Lenz, G. Peters, and A.V. Radyushkin, Phys. Rev. D **73**, 034020 (2006).
- [13] Y. L. Liu and M. Q. Huang, Nucl. Phys. A **821**, 80 (2009).
- [14] Y. L. Liu, M. Q. Huang and D. W. Wang, Eur. Phys. J. C **60**, 593 (2009); Y. L. Liu and M. Q. Huang, Phys. Rev. D **79**, 114031 (2009).
- [15] M.Q. Huang and D.W. Wang, arXiv:hep-ph/0608170.
- [16] M. B. Gavela, Phys. Lett. B **83**, 367 (1979).
- [17] R. Perez-Marcial, R. Huerta, A. Garcia and M. Avila-Aoki, Phys. Rev. D **40**, 2955 (1989); D **44**, 2203(E) (1991).
- [18] R. L. Singleton, Phys. Rev. D **43**, 2939 (1991).
- [19] H. Y. Cheng and B. Tseng, Phys. Rev. D **53**, 1457 (1996); D **55**, 1697(E) (1997).
- [20] S. Migura, D. Merten, B. Metsch and H. R. Petry, Eur. Phys. J. A **28**, 55 (2006).
- [21] H. G. Dosch, E. Ferreira, M. Nielsen and R. Rosenfeld, Phys. Lett. B **431**, 173 (1998); R. S. Marques de Carvalho, F. S. Navarra, M. Nielsen, E. Ferreira and H. G. Dosch, Phys. Rev. D **60**, 034009 (1999).
- [22] V. M. Braun, R. J. Fries, N. Mahnke, and E. Stein, Nucl. Phys. **B589**, 381 (2000); **B607**, 433(E)(2001).
- [23] V. M. Braun, S. E. Derkachov, G. P. Korchemsky and A. N. Manashov, Nucl. Phys. B **553**, 355 (1999).
- [24] B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981); **B191**, 591(E) (1981); Z. Phys. C **18**, 67 (1983); V. M. Belyaev and B. Yu. Blok, Z. Phys. C **30**, 151 (1986); M. A. Ivanov *et al.*, Phys. Rev. D **61**, 114010 (2000); D. W. Wang and M. Q. Huang, *ibid.* D **67**, 074025 (2003).
- [25] V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. B **246**, 52 (1984); V.L. Chernyak, A.A. Ogloblin, I.R. Zhitnitsky, Z. Phys. C **42**, 569 (1989), Yad. Fiz. **48**, 1410 (1988), Sov. J. Nucl. Phys. **48**, 896 (1988).
- [26] L. Wang and F. X. Lee, Phys. Rev. D **78**, 013003 (2008).
- [27] M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij, M. A. Pisarev, and A. G. Rusetsky, Phys. Rev. D **61** (2000) 114010; M. A. Ivanov, V. E. Lyubovitskij, Phys. Rev. D **56** (1997) 348; S. Groote, J. G. Korner, and O. I. Yakovlev, Phys. Rev. D **54** (1996) 3447.
- [28] C. Amsler et al. (Particle Data Group), Phys. Lett. B **667** (2008) 1.
- [29] A. P. Bakulev and A. V. Radyushkin, Phys. Lett. B **271**, 223 (1991); S. V. Mikhailov and A. V. Radyushkin, Phys. Rev. D **45**, 1754 (1992).